LE 2240
Exam \#1
Due by 9:15AM, Tuesday, September 28, 2021
[open book, open notes, calculator and computer allowed - no internet access]
Work must be neat, orderly, and complete in order to receive partial credit. PLEASE submit your solutions as a single PDF file.

1. Use any method to determine the numerical value of $V_{x}$.


$$
\begin{gathered}
I_{1}=\frac{30+V_{x}}{10} \quad I_{2}=\frac{V_{x}-15}{15} \\
I_{1}+I_{2}+2=0 \\
3+\frac{V_{x}}{10}+\frac{V_{x}}{15}-1+2=0 \\
\frac{V_{x}}{6}=-4 \\
V_{x}=-24 \mathrm{~V}
\end{gathered}
$$

Mesh Analysis
Systematic application of KVL.


For mesh $\# 1: \quad 2 I_{1}+10\left(I_{1}-I_{2}\right)-10=0$
For mesh 2: $5\left(I_{2}\right)+3 I_{2}+10\left(I_{2}-I_{1}\right)=0$

$$
\left[\begin{array}{cc}
12 & -10 \\
-10 & 18
\end{array}\right]\left[\begin{array}{l}
I_{1} \\
I_{2}
\end{array}\right]=\left[\begin{array}{c}
10 \\
0
\end{array}\right]
$$

Cramer's Rule:

$$
\begin{aligned}
& I_{1}=\frac{\left|\begin{array}{cc}
10 & -10 \\
0 & 18
\end{array}\right|}{\left|\begin{array}{cc}
12 & -10 \\
-10 & 18
\end{array}\right|}=\frac{(10 \times 18)-(.10 \times 0)}{(12 \times 18)-(-10)(-10)} \\
& =\frac{180}{216-100}=\frac{180}{116} \\
& =\frac{45}{29} \mathrm{~A} \\
& I_{2}=\frac{\left|\begin{array}{cc}
12 & 10 \\
-10 & 0
\end{array}\right|}{116}=\frac{0-(-100)}{116} \\
& =\frac{100}{116}=\frac{25}{29} \mathrm{~A}
\end{aligned}
$$



Mesh 1: $-10+20 I_{1}+20=0$
Mesh 2: $-20+50 I_{2}+30=0$
Mesh 3: $-30+10 I_{3}=0$

$$
\left[\begin{array}{ccc}
20 & 0 & 0 \\
0 & 50 & 0 \\
0 & 0 & 10
\end{array}\right]\left[\begin{array}{l}
I_{1} \\
I_{2} \\
I_{3}
\end{array}\right]=\left[\begin{array}{c}
10-20 \\
20-30 \\
30
\end{array}\right]
$$



$$
\begin{aligned}
& I_{3}=5 \mathrm{~A} \\
& I_{2}-I_{1}=3 \mathrm{~A} \\
& \begin{array}{l}
\text { (constraint equation) } \\
-50+2 I_{1}+4 I_{2}+6\left(I_{2}-I_{3}\right)=0
\end{array} \begin{array}{c}
\text { (constraint equation) } \\
\text { Supermes } 4 \text { ) }
\end{array} \\
& {\left[\begin{array}{ccc}
0 & 0 & 1 \\
-1 & 1 & 0 \\
2 & 10 & -6
\end{array}\right]\left[\begin{array}{c}
I_{1} \\
I_{2} \\
I_{3}
\end{array}\right]=\left[\begin{array}{c}
5 \\
3 \\
50
\end{array}\right]}
\end{aligned}
$$



Super Mesh

$$
\begin{aligned}
& I_{B}-I_{A}=3 A \\
& I_{B}-I_{C}=5 A \\
& {\left[\begin{array}{ccc}
2 & 4 & 6 \\
-1 & 1 & 0 \\
0 & 1 & -1
\end{array}\right]\left[\begin{array}{l}
I_{A} \\
I_{B} \\
I_{C}
\end{array}\right]=\left[\begin{array}{c}
50 \\
3 \\
5
\end{array}\right] }
\end{aligned}
$$



$$
\begin{aligned}
& I_{3}=3 I_{x} \quad \text { (constraint equation) } \\
& -25+10\left(I_{1} \cdot I_{3}\right)+20\left(I_{1} \cdot I_{2}\right)=0 \quad\binom{\text { KVL for }}{\text { mesh }} \\
& 20\left(I_{2}-I_{1}\right)+5\left(V_{y}\right)+50 I_{2}=0 \quad \text { (KVL for mesh } 2 \\
& I_{x}=I_{1}-I_{2} \\
& V_{y}=50 I_{2}
\end{aligned}
$$

In matrix form:

$$
\left[\begin{array}{ccccc}
0 & 0 & 1 & -3 & 0 \\
30 & -20 & -10 & 0 & 0 \\
-20 & 70 & 0 & 0 & 5 \\
-1 & 1 & 0 & 1 & 0 \\
0 & -50 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
I_{1} \\
I_{2} \\
I_{3} \\
I_{x} \\
V_{y}
\end{array}\right]=\left[\begin{array}{c}
0 \\
25 \\
0 \\
0 \\
0
\end{array}\right]
$$

